Zero and Number in UNNS: A Detailed Foundational Note Linking Substrate Vacuum, Modular Structure, and the Multi-Faceted Nature of Number

UNNS Research Notes

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Abstract

This document develops a detailed, self-contained account of two foundational themes in the Unbounded Nested Number Sequence (UNNS) framework: the role of *zero* (the substrate vacuum) and the notion of *number*. We present the UNNS axioms, formalize zero as a universal fixed point, prove stability results, and give a central theorem showing zero's dual role as an absorbing nest and as a universal modulus anchor. We then develop precise, layered definitions of number in UNNS (Number-as-Event, Number-as-Nest, Number-as-Coefficient, Number-as-Echo, Number-as-Perceptual-Form), provide propositions that relate these perspectives, and illustrate with worked examples (Fibonacci, companion matrix, modulus collapse). Two TikZ diagrams visualize (i) the Octad operators acting on the substrate vacuum and (ii) the layered roles of number. The note closes with operational implications for the Octad, an experiment plan, and pseudocode for basic UNNS diagnostics.

Contents

1 Introduction

UNNS places recursion and its operational grammar at the center of a mathematical and computational substrate. Two concepts recur across the UNNS corpus: the *substrate vacuum* (zero) and the notion of *number*. Although superficially simple, both acquire rich structural and operational content in UNNS. This note aims to make those roles explicit, rigorous where possible, and practical for implementation and experimentation.

2 UNNS axioms and notation

We collect the working axioms used through this note and fix notation.

Axiom 2.1 (Recursion)**.** *Every primitive UNNS object is generated by a finite-order recurrence: for each local patch (or motif), there exists an $r \in \mathbb{N}$ and coefficients $c_1, \ldots, c_r$ such that*

$$u_{n+r} = \sum_{j=1}^{r} c_j \, u_{n+r-j} + s_n,$$

where s_n is an optional source term.

Axiom 2.2 (Nesting)**.** *Recurrences embed hierarchically: there is a level index $\ell \geq 0$ and nested lattices Λ_ℓ with spacing a_ℓ so that $\Lambda_{\ell+1}$ refines Λ_ℓ; recurrences at different levels can exchange information (upwards/downwards).*

Axiom 2.3 (Zero (vacuum))**.** *The zero configuration* $\mathbf{0}$ *(all entries equal to 0) is a universal fixed point of homogeneous recurrences and serves as the substrate vacuum and reference baseline for diagnostics.*

Axiom 2.4 (Operators)**.** *An operational octad acts on the substrate: Inletting $\mathcal{I}$, Inlaying $\mathcal{J}$, Repair/Normalization $\mathcal{R}$, Trans–Sentifying $\mathcal{T}$, Branching $\mathcal{B}$, Merging $\mathcal{M}g$, Shadowing $\mathcal{S}$, Projection* Π*. These are primitive actions on $\mathcal{U}$.*

Axiom 2.5 (Invariants)**.** *Certain algebraic constants (e.g. dominant eigenvalues, cyclotomic residues) inevitably arise from the recursion structure and serve as UNNS invariants.*

Notation. We denote a local finite-dimensional state vector by $v \in \mathbb{R}^N$ (or $\mathbb{C}^N$), and local linearized update by $v^{(t+1)} = Cv^{(t)} + f^{(t)}$, where C is the companion / propagation matrix and $f^{(t)}$ the source. The spectral radius is $\rho(C)$.

3 Zero: algebraic fixed-point and spectral stability

We start with algebraic facts about the zero configuration and continue with stability lemmas needed in practice.

Lemma 3.1 (Zero is a fixed point of homogeneous recurrences). *Let $u_{n+r} = \sum_{j=1}^{r} c_j u_{n+r-j}$ be a homogeneous linear recurrence. Then $u_n \equiv 0$ is a solution.*

Proof. Immediate substitution: right-hand side is sum of zeros. □

Remark 3.2. *Lemma 3.1 shows that any operator that preserves linear homogeneity (no forcing) also preserves the vacuum. Inletting or any nonzero source violates homogeneity and therefore moves the system away from* $\mathbf{0}$.

Lemma 3.3 (Spectral contractive stability toward zero). *Let the linear autonomous iteration be $v^{(t+1)} = Cv^{(t)}$. If $\rho(C) < 1$ then $v^{(t)} \to 0$ exponentially for every $v^{(0)}$.*

Proof. Standard: choose a matrix norm $\|\cdot\|$ with induced operator norm; then $\|C^t\| \leq M\rho(C)^t$ for some M, hence $\|v^{(t)}\| \leq M\rho(C)^t \|v^{(0)}\| \to 0$. □

Remark 3.4. *In UNNS applications C is often local and time-dependent; Lemma 3.3 applies to frozen-time linearization and gives a practical diagnostic: if local linearizations have $\rho < 1$, the vacuum is locally attractive.*

4 Zero as Nest and Zero as Modulus: central theorem

We formalize the dual (structural + arithmetic) roles of zero.

Definition 4.1 (Nest). *A* nest *is a hierarchical sequence of motifs / subsequences $\mathcal{N} = \{u^{(\ell)}\}_{\ell \geq 0}$ where each level ℓ is defined by local recurrence rules and an embedding map $\iota_{\ell \to \ell+1}$.*

Definition 4.2 (Zero nest). *The zero nest $\mathcal{N}_0$ is the nest with $u^{(\ell)} \equiv 0$ at every level.*

Definition 4.3 (Modulus anchor). *For integer $m \geq 2$, the zero congruence class $[0]_m$ is the set of integers divisible by m. We say $[0]_m$ is a modulus anchor if it is invariant under the algebraic operations induced by the recurrence coefficients (addition and scalar multiplication by integers).*

Theorem 4.4 (Zero duality — absorbing nest and modulus anchor). *Under the UNNS axioms, zero plays both roles:*

1. *(Absorbing nest) $\mathcal{N}_0$ is absorbing: any collapse mapping of a nest into constant-zero levels yields $\mathcal{N}_0$, and once a local region is in $\mathcal{N}_0$ it remains there under homogeneous updates.*

2. *(Universal modulus anchor) For every modulus $m \geq 2$, the zero class $[0]_m$ is invariant under integer-linear combinations produced by recurrences with integer coefficients; it therefore serves as the universal residue anchor for modular diagnostics.*

Proof. (1) Absorption follows from Lemma 3.1: homogeneous recurrences preserve zero. A collapse mapping (e.g. repair/excision that sets local values to zero) trivially maps to $\mathcal{N}_0$. Since embeddings between levels respect recurrence structure, zero at one level pulls higher-layer states into zero under consistent coupling.

(2) For integer coefficients $c_j \in \mathbb{Z}$, any integer-linear combination of entries that are all multiples of m yields another multiple of m. Thus the subspace of sequences with values in $[0]_m$ is invariant under recurrence updates. Therefore $[0]_m$ anchors modular diagnostics: a sequence is "zero mod m" invariantly if it lies in this class at every step. □

Remark 4.5. *Theorem 4.4 formalizes why zero is both a structural object (nest) and an arithmetic reference (modulus). This duality is exploited in diagnostics (vacuum detection, modular residues) and operator design (repair targets, projection rules).*

5 Operational consequences for the Octad

The Octad operators (Inletting $\mathcal{I}$, Inlaying $\mathcal{J}$, Repair $\mathcal{R}$, Trans–Sentifying $\mathcal{T}$, Branching $\mathcal{B}$, Merging $\mathcal{M}g$, Shadowing $\mathcal{S}$, Projection Π) interact with zero in characteristic ways:

- **Creators** $(\mathcal{I}, \mathcal{J}, \mathcal{B})$ typically move the substrate away from $\mathbf{0}$ by inserting seeds, motifs, or branched copies.
- **Neutralizers/contractors** $(\mathcal{R}, \mathcal{M}g, \Pi)$ often act to push towards $\mathbf{0}$ (proofreading, merging averages, coarse-graining).
- **Translators** $(\mathcal{T}, \mathcal{S})$ either render deviations perceptible or hide them (shadow fields preserve observable vacuum).

Zero thus supplies action semantics: operators are judged by whether they *preserve*, *create*, or *re-interpret* departures from the vacuum.

6 What is a Number in UNNS? — layered formalization

We now give precise definitions for the several senses of "number" in UNNS.

6.1 Number-as-Event

Definition 6.1 (Number-as-Event)**.** *A* number-as-event *is the index or value arising from a canonical generating recurrence at step n; formally, if $u_{n+1} = f(u_n)$ with u_0 given, then $n \mapsto u_n$ is the event stream and u_n is the event expression of the number n in context of the chosen recurrence.*

Remark 6.2. *Different generating recurrences yield different event semantics for the same integer label n. Thus the identity of a number includes the process that generated it.*

6.2 Number-as-Nest (Depth)

Definition 6.3 (Number-as-Nest)**.** *For a nested family $\{\mathcal{N}_\ell\}$, the integer ℓ denoting depth is the number-as-nest; it is an address in the hierarchical lattice.*

Example 6.4. *In a dyadic branching UNNS, level ℓ corresponds to 2^ℓ leaves; the number ℓ indexes nested depth rather than cardinality.*

6.3 Number-as-Coefficient

Definition 6.5 (Number-as-Coefficient)**.** *A number used as a recurrence coefficient* c_j *is interpreted as a structural operator that scales contributions from lower-lag terms in the recurrence; its value quantifies the propagation strength.*

Remark 6.6. *Coefficients determine companion matrix entries; hence they determine spectral properties and long-term growth/decay.*

6.4 Number-as-Echo (Resonance)

Definition 6.7 (Number-as-Echo)**.** *An echo is a constant or limit value arising as an asymptotic invariant of recursion (e.g.* φ *from Fibonacci). Numbers-as-echo are limits or algebraic constants produced by the substrate.*

Example 6.8. *The ratio limit* $\lim_{n\to\infty} F_{n+1}/F_n = \varphi$ *is an echo constant of the Fibonacci recurrence.*

6.5 Number-as-Perceptual-Form

Definition 6.9 (Number-as-Perceptual-Form)**.** *Via Trans–Sentifying* $\mathcal{T}$*, a number can be mapped to an experiential form: a melody, a spatial motif, a glyph. The number-as-perceptual-form is thus the rendering of a numeric invariant into a sensory channel.*

Example 6.10. *Map prime indices to pitch sequences; the perceived pattern identity becomes part of number's meaning.*

7 Relations among the views and formal propositions

We state precise relations linking the perspectives.

Proposition 7.1 (Event $\to$ Echo via spectral data)**.** *Let a linear recurrence have companion matrix* C *with dominant eigenvalue* λ*. If events* u_n *satisfy* $u_n \approx \alpha\lambda^n$ *asymptotically, then the ratio* λ *is a number-as-echo derived from the number-as-event stream.*

Proof. Standard linear recurrence asymptotics: dominant eigenvalue term dominates for large n. □

Proposition 7.2 (Coefficient $\leftrightarrow$ Spectrum)**.** *The collection of numbers used as coefficients determines the companion matrix spectrum, hence the echo constants. Thus number-as-coefficient causally influences number-as-echo.*

Proposition 7.3 (Nest indexing compatible with Projection)**.** *If a projection* Π *commutes with level-embeddings (i.e., respects nesting), then numbers-as-nest (depth indices) are preserved under* Π *up to coarse-graining; otherwise depth information is partially lost.*

Remark 7.4. *These propositions formalize that the different senses of number are not independent: recurrence coefficients set spectra; event streams produce echoes; nest indices may be lost under projection to observables.*

8 Worked examples

8.1 Fibonacci: numbers across views

- Number-as-event: F_n is the n-th Fibonacci number, an output of the recurrence.
- Number-as-nest: n indexes depth in the 1D Fibonacci nesting.
- Number-as-coefficient: coefficients $(1, 1)$ determine propagation.
- Number-as-echo: φ emerges as the asymptotic ratio.
- Number-as-perceptual-form: sonify F_n to pitches: the melodic contour is a perceptual rendition.

8.2 Companion matrix and spectral diagnostics

Given recurrence $u_{n+2} = c_1 u_{n+1} + c_2 u_n$, the companion matrix is

$$C = \begin{pmatrix} c_1 & c_2 \\ 1 & 0 \end{pmatrix}.$$

Its spectral radius $\rho(C)$ determines contraction/expansion. If $c_1 = c_2 = 1$, eigenvalues are $\varphi,\ 1 - \varphi$.

8.3 Modulus collapse example

Take $u_{n+1} = 2u_n$ with $u_0 = 3$ in integers. Modulo 6 the sequence is $3, 0, 0, \ldots$ and collapses immediately into $[0]_6$, illustrating zero-as-modulus anchor.

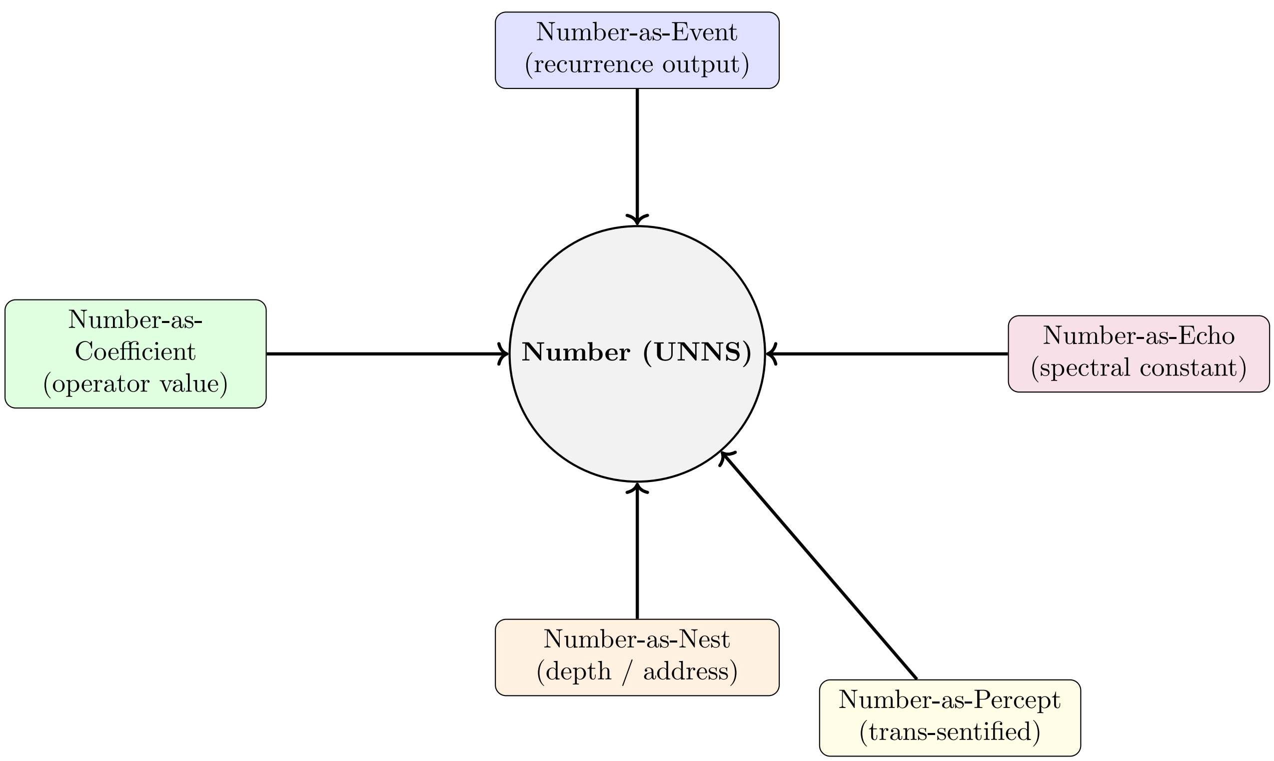
9 Visualization diagrams (TikZ)

Figure 1. Octad operators and the substrate vacuum

$\mathcal{M}g$; [draw, rounded corners, fill=purple!12, left=4.0cm of zero] (T) Trans–Sentifying
$\mathcal{T}$; [draw, rounded corners, fill=gray!12, right=4.0cm of zero] (S) Shadowing
$\mathcal{S}$; [-¿, very thick, blue!70] (I) – (zero); [-¿, very thick, orange!70] (J) – (zero); [-¿, very thick, teal!70] (B) – (zero); [-¿, dashed, very thick, green!60!black] (R) – (zero); [-¿, dashed, very thick, yellow!70!black] (Pi) – (zero); [-¿, dashed, very thick, teal!70] (Mg) – (zero); [-¿, thick, purple!70] (T) – node[midway, above]render / map (zero); [-¿, dashed, thick, gray!70] (S) – node[midway,

above]hide / kernel (zero);

Figure 2. Layered roles of a number in UNNS



10 Practical diagnostics, pseudocode and experiment plan

10.1 Vacuum detection pseudocode

```
# Input: local window values u[i], threshold eps_abs, spectral threshold rho_max
```

```
def is_vacuum(u, eps_abs, C_local):
    if max(abs(u)) < eps_abs:
        if spectral_radius(C_local) < rho_max:
            return True
    return False
```

10.2 Simple experiment plan

1. **Local tests:** construct random recurrences with integer coefficients; measure time to collapse to vacuum under small perturbations; catalogue dependence on spectral radius.

2. **Modular diagnostics:** for each recurrence, compute residues modulo primes and verify invariant classes $[0]_m$ under updates.

3. **Number semantics:** for canonical recurrences (Fibonacci, Tribonacci, linear maps) produce trans-sentified renderings and examine perceptual invariants (spectra, timbre).

4. **Octad workflows:** implement frontend-only prototypes of Inletting / Repair / Shadowing interactions to show operator semantics with respect to vacuum.

11 Discussion and concluding remarks

This master note collects and unifies several foundational observations: zero is both vacuum (nest) and modulus anchor; numbers are multi-faceted in UNNS, being simultaneously events, addresses, operators, echoes, and perceptual forms. These notions are tightly connected: coefficients shape spectra (echoes), which arise from events and inform projection and trans-sentification, and all are measured relative to the vacuum baseline.

UNNS thus supplies a disciplined alternative orientation: axioms are operational and recursive, numbers are processual, and zero is the universal reference. The practical value is that diagnostics and operator design (repair, branching, shadowing) can be expressed in terms of spectral thresholds and modular checks grounded in the theorems above.

A Appendix A: Additional remarks on nonlinearity

All stability lemmas above were linear or linearized. In practice UNNS dynamics can be nonlinear; one may attempt to show Lyapunov stability of zero for nonlinear updates using small-gain conditions or contraction metrics. These are standard extensions but depend on concrete nonlinear forms.

B Appendix B: Selected references and further reading

References

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[3] B. C. Berndt, R. J. Evans, *Sums of Gauss, Eisenstein, Jacobi, Jacobsthal, and Brewer*, (2007).